GEOMAGNETIC FIELD MODELING

Patrick Alken

Outline

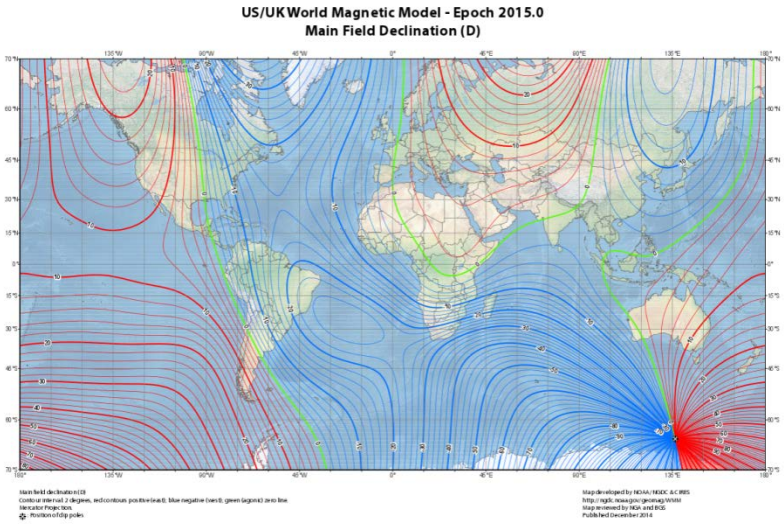
- Introduction to the geomagnetic field
- Magnetic coordinate systems
- Some history of field modeling
- Main field models
- The geomagnetic spectrum

Why do we need geomagnetic models?

- The geomagnetic field has been used for centuries as a natural method of navigation; today mathematical models of the field are used in navigation and heading systems, directional drilling software for natural resources, and smartphones
- The geomagnetic field is constantly changing in unpredictable ways, and so it is necessary to continually update these models to maintain accuracy
- There are many sources of the geomagnetic field which are not well understood, so reseachers constantly attempt to improve models of each source to better understand the field as a whole

World Magnetic Model

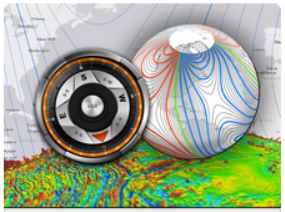
- Operational model
- Joint funded by US/British governments
- Used in many navigation, attitude and heading systems and software
- Provided in many smartphones
- Updated every 5 years
- Most recent release: December 2014



Navigating the world just got easier with the new World Magnetic Model for 2015–2020

December 15, 2014

The Earth's magnetic field has been used for navigation since ancient times. Magnetic navigation has continued to improve alongside transportation technologies so that now magnetic models are used in planes, ships and even in your smartphones. To improve magnetic navigation, NOAA scientists at the National Geophysical Data Center (NGDC) in Boulder, Colorado, have tracked the changing magnetic field using satellites to provide the most accurate and reliable models so that users can navigate their world with ease and precision. The World Magnetic Model (WMM) is NGDC's newest spherical harmonic model of the magnetic field produced by the Earth's core and will accurately represent the main magnetic field on Earth for the next five years.



One degree declination contours across the globe. These lines indicate the difference between where a compass points the compass direction and geographic north. Red lines are for negative declination, blue lines are positive and green indicates zero degrees of declination.

Introduction to the World Magnetic Model

The WMM is a large-scale representation of Earth's magnetic field that gives analog and digital magnetic compasses dependable accuracy. It is used extensively in smartphone apps as an accurate way to know which way the user is going.

Using years worth of satellite data, NOAA's NGDC are able to model Earth's changing magnetic field and predict what it will look like over the next five years. Produced every five years since 1975, this model of the magnetic field produced by Earth's rapidly spinning metallic core has been critical in aiding navigation for the military, scientific exploration, nautical charting and citizen transportation. Since 2005, NGDC has worked closely with the British Geological Survey (BGS) to provide the most accurate models that are used for the myriad of navigational uses across the globe.

The World Magnetic Model continues to be the navigation standard model for the entire U.S. Department of Defense (DOD), the North Atlantic Treaty Organization (NATO), International Hydrographic Organization (IHO), Federal Aviation Administration (FAA), and consumer electronics (e.g., iOS, Blackberry, Android) and many others.

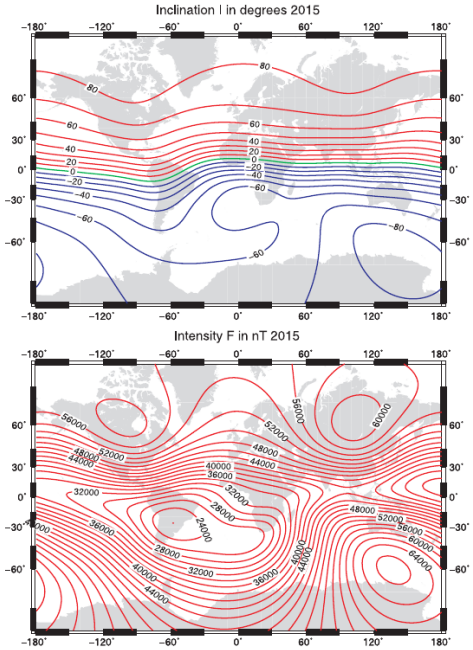
Online Magnetic Calculators

NGDC provides multiple online tools and softwares to estimate current and past values of the magnetic field:

- **Declination Calculator:** magnetic declination (variation) for a single day anytime between 1590–present.

International Geomagnetic Reference Field

- Research model collaboration between many international partners
- Used in many areas of geomagnetic research
- Updated every 5 years



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International Geomagnetic Reference Field: the 12th generation

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Abstract

The 12th generation of the International Geomagnetic Reference Field (IGRF) was adopted in December 2014 by the Working Group V-MOD appointed by the International Association of Geomagnetism and Aeronomy (IAGA). It updates the previous IGRF generation with a definitive main field model for epoch 2010.0, a main field model for epoch 2015.0, and a linear annual predictive secular variation model for 2015.0-2020.0. Here, we present the equations defining the IGRF model, provide the spherical harmonic coefficients, and provide maps of the magnetic declination, inclination, and total intensity for epoch 2015.0 and their predicted rates of change for 2015.0-2020.0. We also update the magnetic pole positions and discuss briefly the latest changes and possible future trends of the Earth's magnetic field.

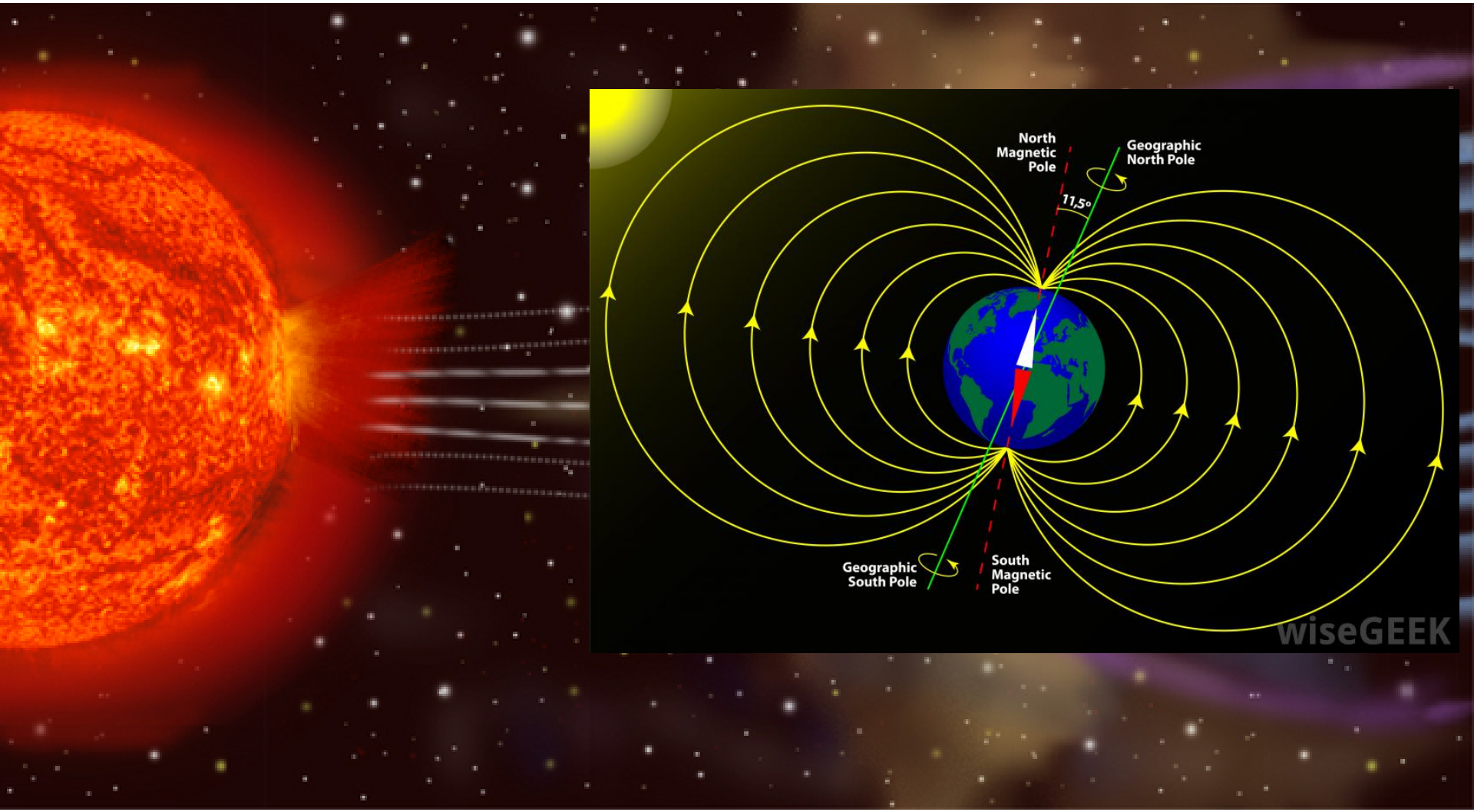
Keywords: Geomagnetism; Field modeling; IGRF

Correspondence/Findings

Introduction

satellites. The IGRF is used by scientists in a wide variety of studies, for instance, concerning the dynamics of

The geomagnetic field

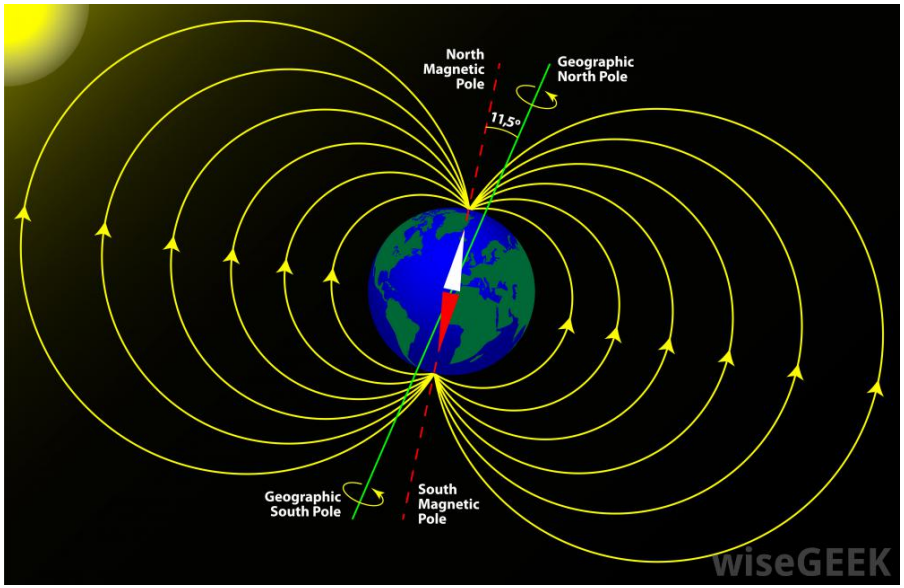


Dipole coordinates

$$\mathbf{B} = \frac{M}{4\pi r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\mathbf{B} = -\nabla V$$

$$V = \frac{M}{4\pi}\frac{\cos\theta}{r^2}$$



How to define a coordinate system in this geometry?

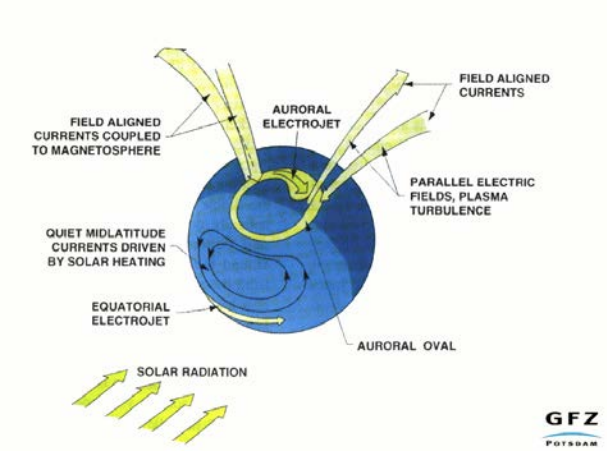
$$q = \frac{\cos\theta}{r^2}, p = \frac{r}{\sin^2\theta}, \phi = \phi$$

Other geomagnetic coordinate systems

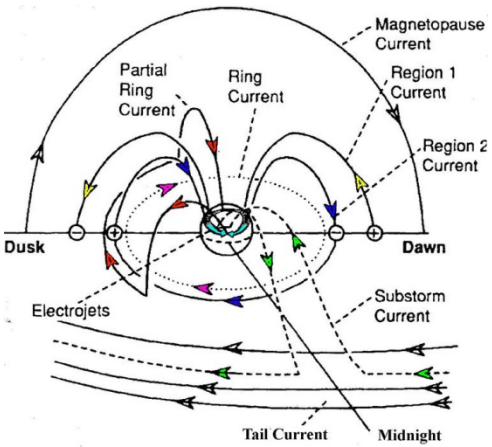
Coordinate system	Reference
Magnetic Apex	VanZandt et al, 1972
Corrected Geomagnetic	Mayaud, 1960
Modified Apex	Richmond, 1995
Quasi-Dipole	Richmond, 1995
Solar Magnetic	Olson, 1970
Geocentric Solar Magnetospheric	Olson, 1970

Geomagnetic Field Sources

$$\mathbf{B}_{measured}(\mathbf{r},t) = \mathbf{B}_{main}(\mathbf{r},t) + \mathbf{B}_{crust}(\mathbf{r}) + \boxed{\mathbf{B}_{ionosphere}(\mathbf{r},t)} + \boxed{\mathbf{B}_{magnetosphere}(\mathbf{r},t)} + \mathbf{B}_{induced}(\mathbf{r},t) + \mathbf{B}_{oceans}(\mathbf{r},t) + \mathbf{B}_{noise}(\mathbf{r},t)$$



$$\mathbf{B}_{EEJ}(\mathbf{r},t) + \mathbf{B}_{Sq}(\mathbf{r},t) + \mathbf{B}_{PEJ}(\mathbf{r},t) + \mathbf{B}_{grav}(\mathbf{r},t) + \mathbf{B}_{pg}(\mathbf{r},t) + \mathbf{B}_{FAC}(\mathbf{r},t)$$



$$\mathbf{B}_{RC}(\mathbf{r},t) + \mathbf{B}_{CT}(\mathbf{r},t) + \mathbf{B}_{MP}(\mathbf{r},t) + ...$$

Some history

Hist. Geo Space Sci., 5, 11–62, 2014
www.hist-geo-space-sci.net/5/11/2014/
doi:10.5194/hgss-5-11-2014
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History of Geo- and Space

Gauss,

Carl Friedrich Gau
***Magnetism* – a revi**

K.-H.

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[2]Jet Propulsion Laboratory,

Correspondence t

Received: 25 July 2013

Abstract. This is a translation of the A
Gauss in 1839 in the *Resultate aus den*

then $P^{(n)}$ has the form of

$$P^{(n)} = g^{n,0}\,P^{n,0} + (g^{n,} + (g^{n,} + \text{etc.}$$

where $g^{n,0}, g^{n,1}, h^{n,1}, g^{n,2},$
specified.

[1]Translators' footnote (footnotes by the translators
with a capital T to discriminate them from the origina
Gauss): a digital version of the original book in Germa
from Google Books.

Published by Copernicus Publications.

The numerical values of the 24 coefficients obtained by the first calculation, counting the longitude λ east of Greenwich, are as follows:

$g^{1,0}$	=	+925.782	$g^{2,2}$	=	+0.493
$g^{2,0}$	=	−22.059	$g^{3,2}$	=	−73.193
$g^{3,0}$	=	−18.868	$g^{4,2}$	=	−45.791
$g^{4,0}$	=	−108.855	$h^{2,2}$	=	−39.010
$g^{1,1}$	=	+89.024	$h^{3,2}$	=	−22.766
$g^{2,1}$	=	−144.913	$h^{4,2}$	=	+42.573
$g^{3,1}$	=	+122.936	$g^{3,3}$	=	+1.396
$g^{4,1}$	=	−152.589	$g^{4,3}$	=	+19.774
$h^{1,1}$	=	−178.744	$h^{3,3}$	=	−18.750
$h^{2,1}$	=	−6.030	$h^{4,3}$	=	−0.178
$h^{3,1}$	=	+47.794	$g^{4,4}$	=	+4.127
$h^{4,1}$	=	+64.112	$h^{4,4}$	=	+3.175.

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),

Modeling approaches

Comprehensive approach

- Try to model all sources simultaneously and extract the source we are interested in
- Results in a very complicated model, with many parameters
- Lots of regularization needed to minimize crosstalk between different sources
- See: Sabaka et al, CM5, a pre-Swarm comprehensive geomagnetic field model derived from over 12 yr of CHAMP, Ørsted, SAC-C and observatory data Geophys. J. Int. (2015) 200 (3): 1596-1626 doi:10.1093/gji/ggu493

Dedicated approach

- Try to remove unwanted signals in the measurements as much as possible, and model what remains
- To remove ionospheric sources, choose night-time data
  - Works well at low/mid latitudes
  - At high latitudes, ionospheric currents flow in sunlight and darkness!
- To remove magnetospheric influence, select geomagnetically quiet periods using geomagnetic indices (there is always a magnetospheric field though!)

Mathematical parameterization

$$V(r,\theta,\phi,t) = a\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}\left(g_{nm}(t)\cos m\phi + h_{nm}(t)\sin m\phi\right)P_{nm}(\cos\theta)$$

Unknown Gauss coefficients to be estimated by inverting magnetic observations

Mathematical parameterization (cont'd)

$$\mathbf{B}(r,\theta,\phi,t) = -\nabla V(r,\theta,\phi,t)$$

$$= \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+2}\left\{\begin{array}{l} g_{nm}(t)\begin{bmatrix}\cos m\phi\partial_\theta \\ \frac{m}{\sin\theta}\sin m\phi \\ -(n+1)\cos m\phi\end{bmatrix} + \\ h_{nm}(t)\begin{bmatrix}\sin m\phi\partial_\theta \\ \frac{-m}{\sin\theta}\cos m\phi \\ -(n+1)\sin m\phi\end{bmatrix}\end{array}\right\} P_{nm}(\cos\theta)$$

$$= \sum_{n=1}^{\infty}\sum_{m=0}^{n} g_{nm}(t)\mathbf{G}_{nm}(r,\theta,\phi) + h_{nm}(t)\mathbf{H}_{nm}(r,\theta,\phi)$$

Mathematical parameterization (cont'd)

Taylor series for time dependence

$$g_{nm}(t) = g_{nm}(t_0) + \dot{g}_{nm}(t - t_0) + \frac{1}{2}\ddot{g}_{nm}(t - t_0)^2$$

$$h_{nm}(t) = h_{nm}(t_0) + \dot{h}_{nm}(t - t_0) + \frac{1}{2}\ddot{h}_{nm}(t - t_0)^2$$

Basis splines for time dependence

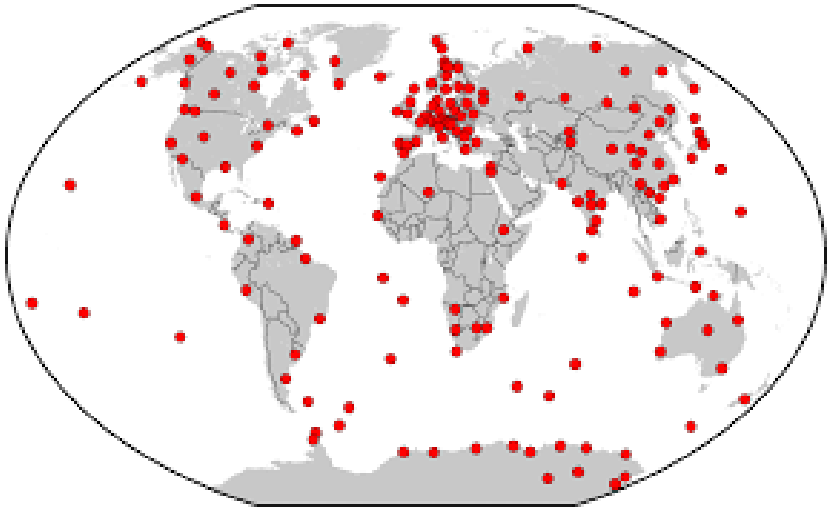
$$g_{nm}(t) = \sum_k g_{nmk} M_k(t)$$

$$h_{nm}(t) = \sum_k h_{nmk} M_k(t)$$

Data sources

Satellites

- Oersted (1999-present)
- SAC-C (1999-2003)
- CHAMP (2000-2010)
- Swarm (2013-present)
- Very good at resolving spatial variations
- Difficulty in resolving temporal variations



Ground observatories

- Global network for many decades
- Very good at resolving temporal variations
- Less good at resolving spatial variations with sparse coverage

The Inverse Problem

$$\mathbf{b} = \mathbf{A}\mathbf{x}$$

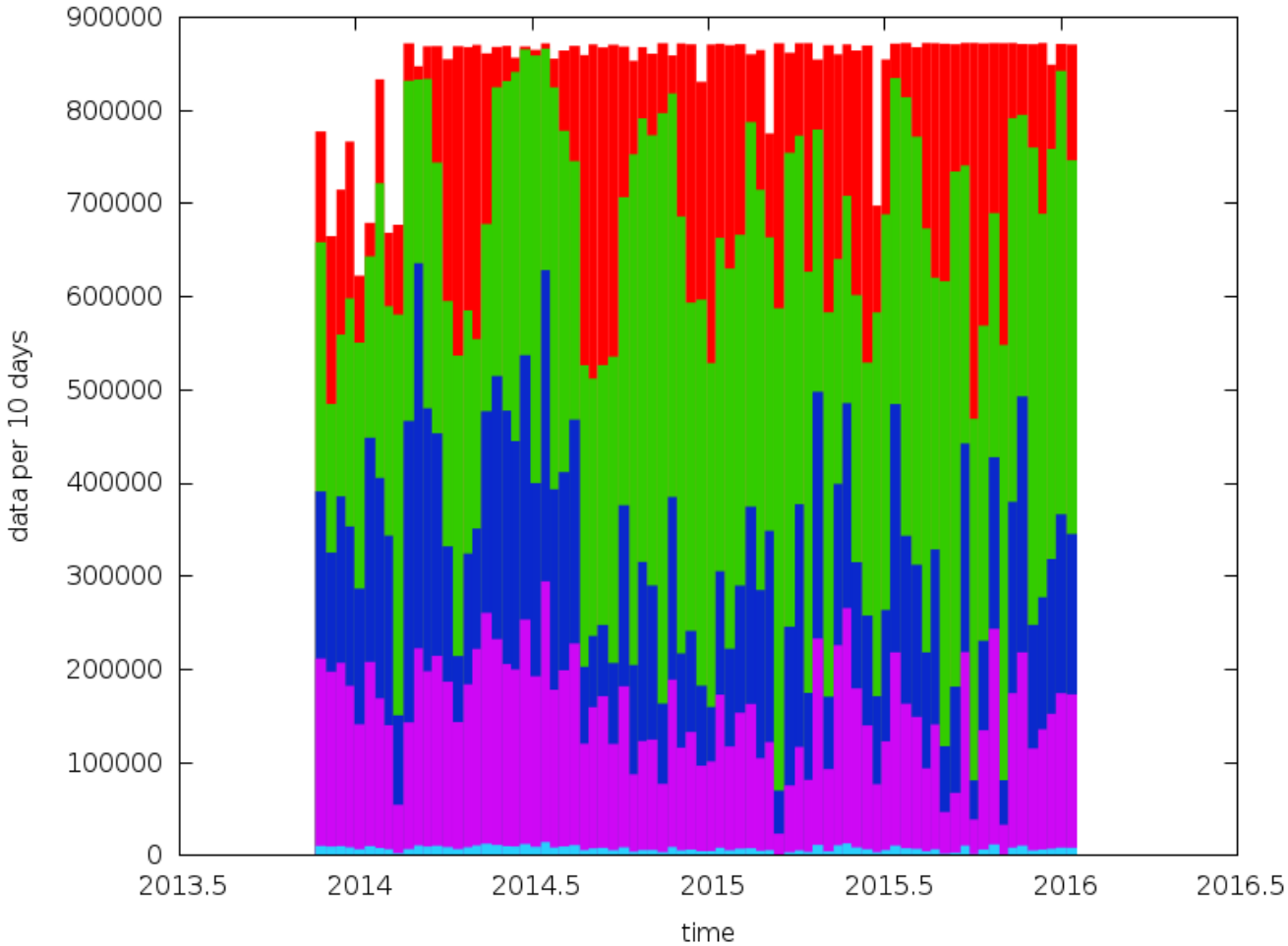
Minimize:

$$\chi^2(\mathbf{x}) = ||\mathbf{b} - \mathbf{A}\mathbf{x}||^2 + \lambda^2||\mathbf{L}\mathbf{x}||^2$$

Data selection

- Time period: 26 November 2013 to 21 January 2016
- Obvious outliers and instrument errors
- Local time: 22:00 to 05:00 (to eliminate ionospheric field at low/mid latitudes)
- Geomagnetically quiet times: |Dst| < 30 nT (to avoid large external disturbance fields)

Data selection (cont'd)



Total data

After removing instrument errors and large external fields

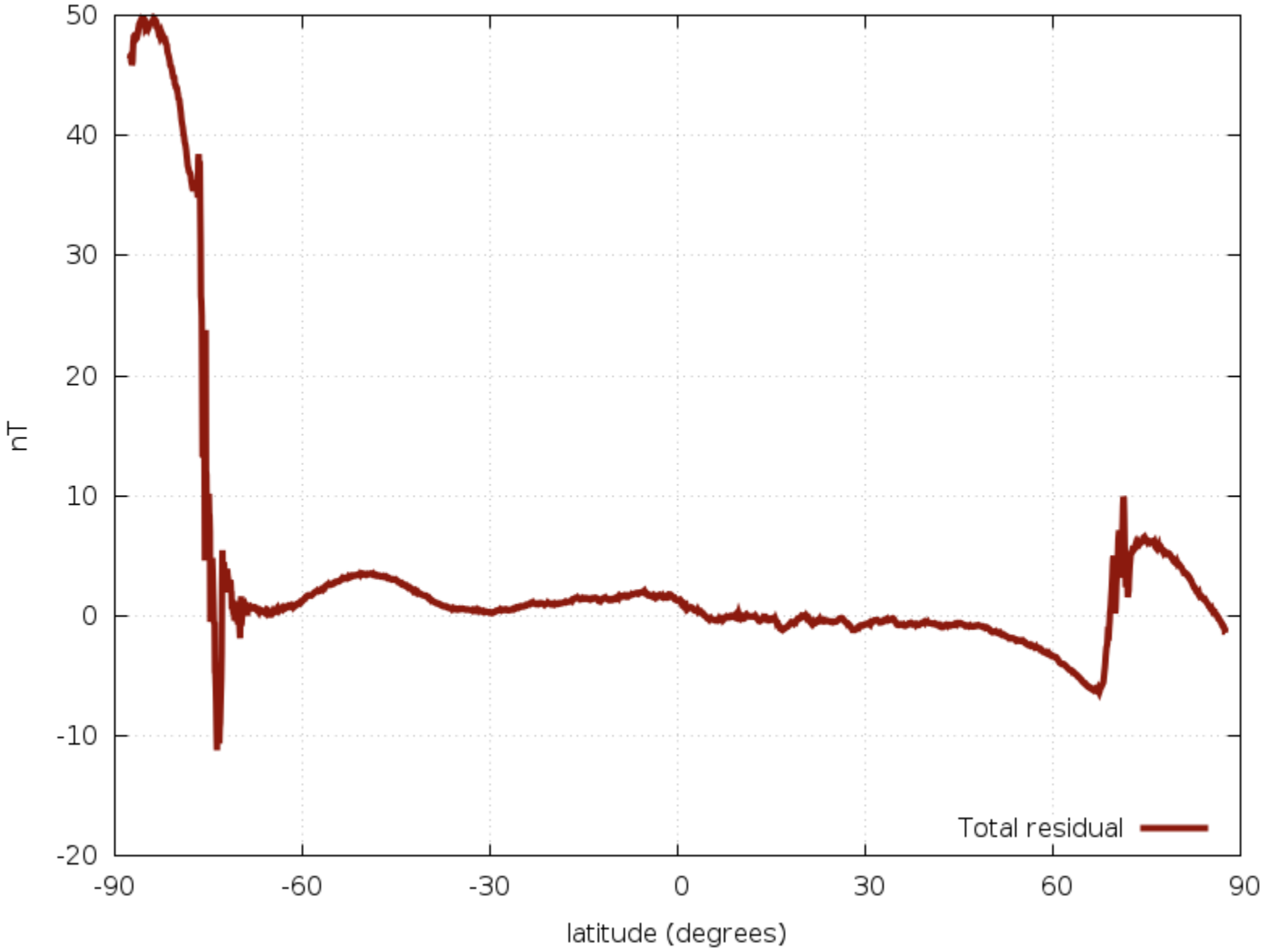
After selecting for quiet periods

After selecting for local time

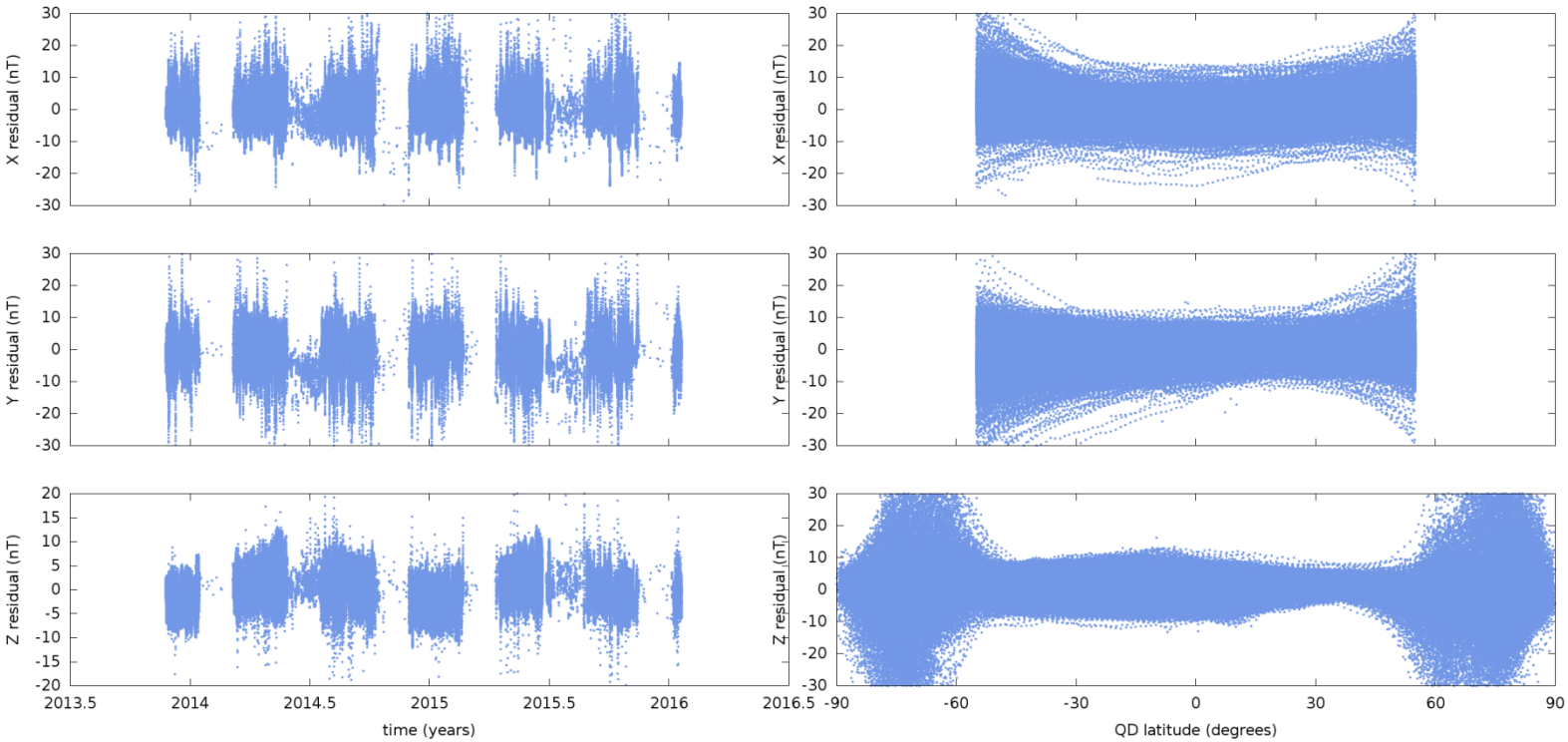
After downsampling by factor 20

99.1% of the data is thrown out!

Single track example (Swarm)



Residuals

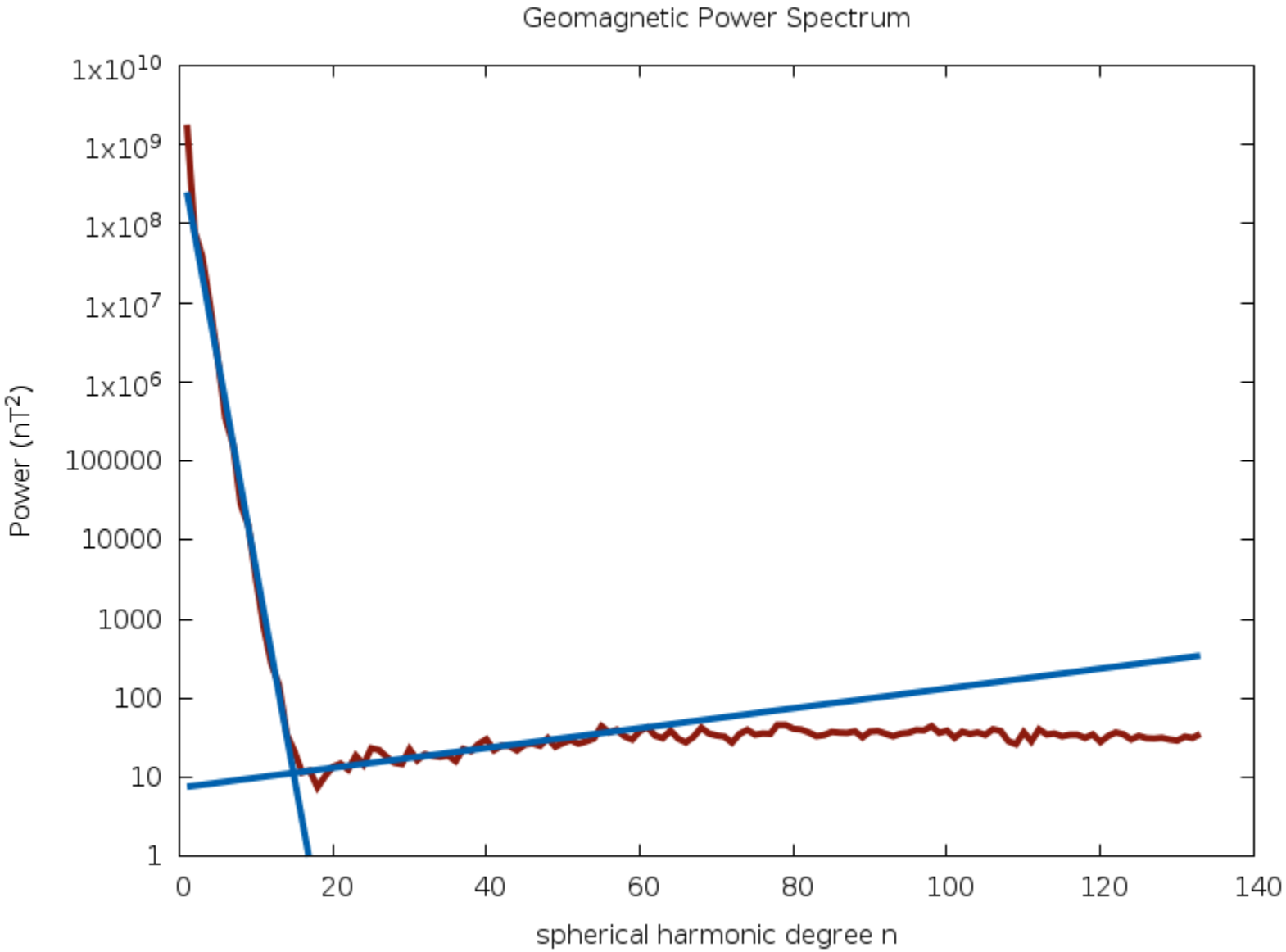


Swarm A	N	mean (nT)	rms (nT)
X	257092	0.15	4.65
Y	257092	-0.54	4.77
Z	430206	0.27	8.60
Low Z	257092	0.29	3.40

Geomagnetic Power Spectrum

$$R_n = (n+1)\sum_{m=0}^{n}\left(g_{nm}^2 + h_{nm}^2\right)$$

Power in the geomagnetic field at the Earth's surface for spherical harmonic degree n

F. Lowes, 1974

Geomagnetic Power Spectrum (cont'd)